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$$+ \frac{2(2b^2c^2 + 2a^2b^2 - 3a^2c^2 - b^4)m^2}{(a^2 - b^2)(b^2 - c^2)(C^2 - 2a^2 - 2b^2 - 2c^2)} \\ + \frac{2(c^4 - 2a^2c^2 - 2b^2c^2 + 3a^2b^2)n^2}{(a^2 - c^2)(b^2 - c^2)(C^2 - 2a^2 - 2b^2 - 2c^2)} = 1.$$

$\therefore l^2/R^2 + m^2/S^2 + n^2/T^2 = 1$ , a co-axial ellipsoid.

### CALCULUS.

89. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

Integrate the equation,  $\frac{dy}{dx} + y \cos x = \frac{\sin 2x}{2}$ .

I. Solution by Dr. E. D. ROE, Jr., Norwood, Mass.

Put  $\sin x - y = z$ ; then  $\cos x dx - dy = dz$ ,  $dy = \cos x dx - dz$ , and the equation becomes

$$\frac{dz}{dx} + (z-1) \cos x = 0, \text{ or } \frac{dz}{z-1} + \cos x dx = 0.$$

Integrating this,  $\log(z-1) + \sin x + \mu = 0$ ,  $z-1 = e^{-\sin x - \kappa}$ ,

$$\text{or } \sin x - y - 1 = e^{-\sin x - \kappa} = ce^{-\sin x}, \quad y = \sin x - 1 + ce^{-\sin x}.$$

Dr. Roe also furnished a second solution.

II. Solution by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, O.

If the equation  $\frac{dy}{dx} + y \cos x = 0$  is solved, the result is  $y = C_1 e^{-\sin x}$ . After substituting in the original equation  $\frac{dC_1}{dx}$  is found to equal  $e^{\sin x} \sin x \cos x$ ; therefore,  $C_1 = e^{\sin x} \sin x - e^{\sin x} + C$ . And  $y = \sin x + Ce^{-\sin x}$ .

III. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass., and JOHN R. JEFFERY, Student in Ohio State University, Columbus, O.

The general form of this equation is  $\frac{dy}{dx} + Py = Q$  of which the general integral is  $e^{\int P dx} y = \int e^{\int P dx} Q dx + C$ . Here  $\int P dx = \int \cos x dx = \sin x$ .

$$\therefore e^{\sin x} y = \int e^{\sin x} \cos x \sin x dx + C.$$

Integrating right member by parts,  $e^{\sin x} y = \sin x \cdot e^{\sin x} - e^{\sin x} + C$ , or  $y = \sin x - 1 + ce^{-\sin x}$ .

[See Johnson's *Differential Equations*, page 35, ex. 7.]

IV. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics in Manual Training School, Philadelphia, Pa.; M. C. STEVENS, M. A., Professor of Mathematics, Purdue University, Lafayette, Ind.; and BURKE SMITH, Senior Class, University of Washington, Seattle, Wash.

$$\frac{dy}{dx} + y \cos x = \sin x \cos x.$$

Multiply by  $e^{\sin x}$  as an integrating factor.

$$\text{Then } e^{\sin x} dy + y e^{\sin x} \cos x dx = e^{\sin x} \sin x \cos x dx.$$

$$\text{Integrating, } y e^{\sin x} = \int e^{\sin x} \sin x \cos x dx.$$

Integrate right hand member by parts, and we have,

$$y e^{\sin x} = e^{\sin x} \sin x - \int e^{\sin x} \cos x dx = e^{\sin x} \sin x - e^{\sin x} + c.$$

$$\therefore y = \sin x - 1 + ce^{-\sin x}.$$

V. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let  $\sin x = \log v$ ; so that  $\cos x dx = dv/v$ . Then the equation becomes  $dy/dv + y/v = \log v/v$ .

$$\therefore v dy + y dv = \log v dv. \therefore vy = v(\log v - 1) + C. \therefore y + 1 = \log v + C/v.$$

$$\therefore y + 1 = \sin x + Ce^{-\sin x}$$

VI. Solution by J. SCHEFFER, A. M., Hagerstown, Md., and ARTHUR C. LUNN, University of Chicago, 5630 Drexel Ave., Chicago, Ill.

Solving first the differential equation  $\frac{\partial y}{\partial x} + y \cos x = 0$ , we find  $y = Ce^{-\sin x}$ ;

differentiating, we have  $\frac{\partial y}{\partial x} = -Ce^{-\sin x} \cos x + e^{-\sin x} \frac{\partial C}{\partial x}$ .

$$\therefore e^{-\sin x} \frac{\partial C}{\partial x} = \frac{1}{2} \sin 2x. \therefore C = \frac{1}{2} \int e^{\sin x} \sin 2x dx.$$

Putting  $\sin x = z$ ,  $\therefore \cos x = \sqrt{1-z^2}$ ,  $dx = \frac{dz}{\sqrt{1-z^2}}$ ; we have,

$$C = \int e^z \cdot z dz = \int z \partial(e^z) = ze^z - \int e^z \partial z = ze^z - e^z = \sin x \cdot e^{\sin x} - e^{\sin x} + c.$$

Substituting this in  $y = Ce^{-\sin x}$ , we obtain  $y = \sin x - 1 + ce$ .

The method employed is Lagrange's method of variation of parameters.

Also solved by HENRY HEATON, and P. H. PHILBRICK.